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ESTIMATION PROPERTIES AND APPLICATION OF EXPONENTIAL PERKS DISTRIBUTION

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ABSTRACT

This study proposed a new extension of perks distribution by adding one shape and one scale parameter to the perk's distribution using the exponential G-family proposed by Bourguignon (2014). This study has derived some expressions for its basic statistical properties such asmoments, moment generating function, the characteristics function, reliability analysis, quantile function and the distribution of order statistics. Some plots of the distribution and the reliability function were generated and interpreted appropriately. The model parameters have beenestimated using the method of maximum likelihood estimation. The performance of the model has also been tested by simulateddata. The distribution compared to some existinggeneralizations ofthe Perks distribution.The likelihood approach is used to estimate the parameters. The efficiency and consistency of these estimators are validated by Monte Carlo simulation analysis, which shows that bias, variance, and root mean squareerror (RMSE) all decline with increasing sample sizes. Applying the EPD to areal-worlddataset; the daily COVID-19-related mortality in China, validates its applicability.According to Kolmogorov-Smirnov statistics, the Bayesian Information Criterion(BIC), the Akaike Information Criterion (AIC), and other well-knowncompetingmodels, the EPD exhibits better goodness-of-fit in both situations. The findingsdemonstrate that the EPD is a reliable and adaptable technique for modelingpositively skewed lifetime and survival data, with special potential for use inreliability analysis, medical research, and epidemiology.

KEYWORDS: Exponentiated Perks distribution, Weibull distribution, Maximum Likelihood, Monte Carlo, weibull distribution, Quartile function.

INTRODUCTION

A probability distribution is a statistical function that describes all the possiblevalues andlikelihoods that a random variable can take within a given range.This range will be bounded between the minimum and maximum possible values.However, where the possible value is likely to be plotted on the probabilitydistribution depends on several factors. These factors include the distribution’smean (average), standard deviation, skewness, and kurtosisHayes (2024). Alshanbariet al. (2023) Generators of distributions is not a new thing, the idea can betraced right from the development of family of exponentiated G by Gupta andKundu (1999) which needed an additional parameters to the distribution.

The Exponential distribution is used in Poisson processes and gives a description of the time between events. It is applied mostly in life testing experiments.It has memoryless property with a constant failure rate which makes the distributionunsuitable for real life problems and hence creating a vital problem instatistical modeling and applications.Besides the applications of Exponential distribution and its attractive properties,its usage has been limited in modeling real life situations due to the fact thatit has a constant failure rate Lemonte (2013) One other problem with exponentialdistribution is in its memoryless property, this is because the memoryless assumptionis hardly obtained in real life situations.Adegoke et al. (2018)proposed

The Perks distribution has several applications in the field of actuarial science. Haberman and Renshaw (2011) and Richards (2008) have shown that this distribution is a good fit for pensioner mortality data. The parametric mortality projection is well-described by the Perks distribution, according to Haberman and Renshaw (2011).

If a random variable X has exponential distribution with parameter $\lambda > 0$*, then, the* CDF and PDF are given by

$$F(x) = 1 - e^{-\lambda x} \tag{1}$$

And

$$f(x) = \lambda e^{-\lambda x} \tag{2}$$

where,$x > 0$, $\lambda > 0$, *and* λ is the scale parameter.

2 The Perks Distribution, PD

Perks distribution was developed by Perks (1932) and it is an extension or modification of the Gompertz-Makeham distribution. The CDF and the PDF of a randomvariable T following perks distribution are define as:

$$G(x) = 1 - \frac{1+\beta}{1+\beta e^{\theta t}} \quad (3)$$

And

$$g(x) = \beta\theta e^{\theta t} \frac{1+\beta}{\left(1+\beta e^{\theta t}\right)^2} \quad (4)$$

3 The exponential G. family of distribution

The Exponential-G family of distributions proposed by Bourguignon et al. (2014) With the CDF and the PDF defined as follows:

$$F(x) = 1 - exp\left\{-\lambda\left[\frac{G(x)}{1-G(x)}\right]\right\} \quad (5)$$

And

$$f(x) = \lambda g(x)\left(1-G(x)\right)^{-2} exp\left\{-\lambda\left[\frac{G(x)}{1-G(x)}\right]\right\} \quad (6)$$

4 The Exponential-Perks Distribution.

The CDF of the EPDcan be obtained by substituting equation (3) in (5)

$$\mathrm{F(x)} = 1 - exp\left\{-\lambda\left[\frac{\beta\left(e^{\theta t}-1\right)}{1+\beta}\right]\right\} \quad (7)$$

While substituting G(x) and g(x)in (6) gives the pdf of the Exponential perks distribution as

$$\mathrm{Pdf} = \frac{\lambda\left(\beta\theta e^{\theta t}\right)}{1+\beta} \exp\left\{-\lambda\left[\frac{-\lambda\beta\left(e^{\theta t}-1\right)}{1+\beta}\right]\right\} \quad (8)$$

5 Properties of EPD

The moment

$$\mu = E[T^n] = \int_0^\infty t^n f(t)dt \quad \textbf{(9)}$$

$$\int_0^\infty t^n f(t)dt = \int_0^\infty \frac{\lambda\beta\theta e^{\theta t}}{1+\beta} e^{-\lambda\beta\left(\frac{e^{\theta t}-1}{1+\beta}\right)} \mathrm{dt} \quad \textbf{(10)}$$

let u= $e^{\theta t}, so\ dt = \frac{du}{\theta u}\quad t = \frac{in\ u}{\theta}$

$$\mu = \int_1^{\infty} (\frac{in\ u}{\theta})^n\ exp[\frac{-\lambda\beta(u-1)}{1+\beta}]\,du \qquad (11)$$

moment generating function

$$mgf(t) = E[e^{xn}] = \int_0^{\infty} e^{tx} f(t)dt \qquad (12)$$

$$M_x = \int_0^{\infty} e^{tx} \frac{\lambda\beta\theta e^{\theta t}}{1+\beta} e^{-\lambda\beta(\frac{e^{\theta t}-1}{1+\beta})}\ dt \qquad (13)$$

Let y = $e^{\theta x}, x = \frac{in\ y}{\theta}$ **and dx =**$\frac{dy}{\theta}$,

$$M_x = \int_1^{\infty} \left(\frac{in\ y}{\theta}\right)\frac{\lambda\beta\theta y}{1+\beta}\ exp\left[-\frac{\lambda\beta(y-1)}{1+\beta}\right]\frac{dy}{d\theta} = \frac{\lambda\beta}{1+\beta}\int_1^{\infty} y^{\frac{t}{\theta}}\ exp[-\lambda\beta\frac{(y-1)}{1+\beta}]dy$$

$$M_x = \frac{\lambda\beta}{1+\beta}\ exp\left(\frac{\lambda\beta}{1+\beta}\right)\left(\frac{1+\beta}{\lambda\beta}\right)^{\frac{t}{\theta}+1}\Gamma(\frac{t}{\theta}+1) \qquad (14)$$

Survival function

This is the probability that an entity or a person won't malfunction after a specified amount of time.Therefore,is given by:

$$S(t) = 1 - F(t) \qquad (9)$$

$$s(t) = 1 - [1 - e^{\{-\lambda\frac{\beta(e^{\theta t}-1)}{1+\beta}\}}] = e^{\{-\lambda\frac{\beta(e^{\theta t}-1)}{1+\beta}\}} \qquad (10)$$

Hazard Function

This refers to the probability or tendency that a given component or system may die over certain period oftime. Mathematically,

$$h(t) = \frac{f(t)}{s(t)} = \frac{\lambda\beta\theta e^{\theta t}}{1+\beta} e^{[\frac{-\lambda\beta(e^{\theta t}-1)}{1+\beta}]} \Big/ e^{\frac{-\lambda\beta(e^{\theta t}-1)}{1+\beta}} \qquad (11)$$

Quantile Function

$Q_u = F^{-1}$

Let f(t)= u, so that $u = 1 - \exp\{\frac{-\lambda\beta(e^{\theta t}-1)}{1+\beta}\}$

$$t = \frac{1}{\theta}\ln[1 - \frac{1+\beta}{\lambda\beta}\ln(1-u)] \qquad (12)$$

ESTIMATION OF PARAMETERS

The parameters are estimated using the maximum likelihood method as follows: The likelihood function is:

$$L(\theta, \lambda, \beta) = \prod_{i=1}^{n} f(x) = \prod_{i=1}^{n} \frac{\lambda\beta\theta e^{\theta x}}{1+\beta} \exp\left[-\frac{\lambda\beta(e^{\theta x}-1)}{1+\beta}\right]$$

$$L(\lambda,\beta,\theta) = in\ L(\lambda,\beta,\theta) = n\ in\ \lambda + n\ in\ \beta + n\ in\theta + \theta\sum_{i=1}^{n} x - n\ in(1+\beta) - \frac{\lambda\beta}{1+\beta}\sum_{i=1}^{n}(e^{\theta x}-1)$$

1. With respect to λ

$$\frac{dl}{d\lambda} = \frac{n}{\lambda}$$

2. With respect to β
3. With respect to θ

Simulation Results

The simulation results are presented in the following Tables and the graphicaldisplay in Figure shows the mean estimates, bias, variance, and RMSE foreach parameter across different sample sizes.The simulation results reveal several important patterns regarding the performance

of the MLEs for the EPD parameters.

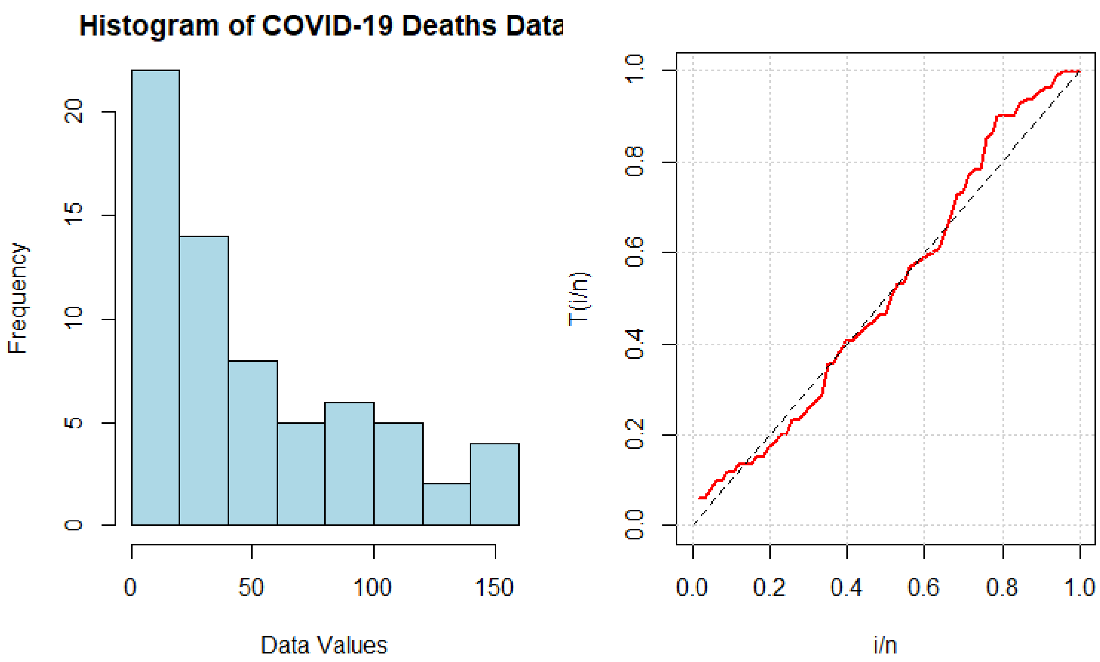
Sample Size	Parameter	Mean Estimate	Bias	Variance	RMSE
25	λ	2.8577	0.3577	6.3399	2.5432
	β	2.7529	2.7529	0.5853	0.7668
	θ	1.0889	0.2889	0.4132	0.7048
50	λ	2.6985	0.1985	2.9722	1.7354
	β	2.7642	0.0642	0.2856	0.5382
	θ	0.9437	0.1437	0.1757	0.4431
75	λ	2.7717	0.608	0.5391	0.7368
	β	2.7608	0.0608	0.5391	0.7368
	θ	0.9108	0.1108	0.1037	0.3406
100	λ	2.5348	0.0348	1.1979	1.0950
	β	2.7030	0.0030	0.1767	0.4203
	θ	0.8836	0.0836	0.0687	0.2751
250	λ	2.5255	0.0255	0.5108	0.7151
	β	2.7112	0.0112	0.0793	0.2819
	θ	0.8327	0.0327	0.0237	0.1574
500	λ	2.5867	0.0867	0.2719	0.5286
	β	2.7099	0.0099	0.1018	0.3192
	θ	0.8042	0.0042	0.0143	0.1197

Data Analysis

The dataset, originally published by Chipepa et al. (2022) Chipepa et al. (2022),documents the daily fatalities attributed to COVID-19 in China during the periodspanning January 23 to March 28, 2020. The recorded values are presented asfollows:

8, 16, 15, 24, 26, 26, 38, 43, 46, 45, 57, 64, 65, 73, 73, 86, 89, 97, 108, 97, 146,
121, 143, 142, 105, 98, 136, 114, 118, 109, 97, 150, 71, 52, 29, 44, 47, 35, 42, 31,
38, 31, 30, 28, 27, 22, 17, 22, 11, 7, 13, 10, 14, 13, 11, 8, 3, 7, 6, 9, 7, 4, 6, 5, 3, 5.

The descriptive statistics presented in Table 4.3 where a median of 33 deathsis pulled up to a mean of 49.7 by extreme values like the maximum of 150. Thedata has right-skeweddistribution as evidently seen in Figure 4.5 while the TTTplot suggests the data to have bathtub failure rate.



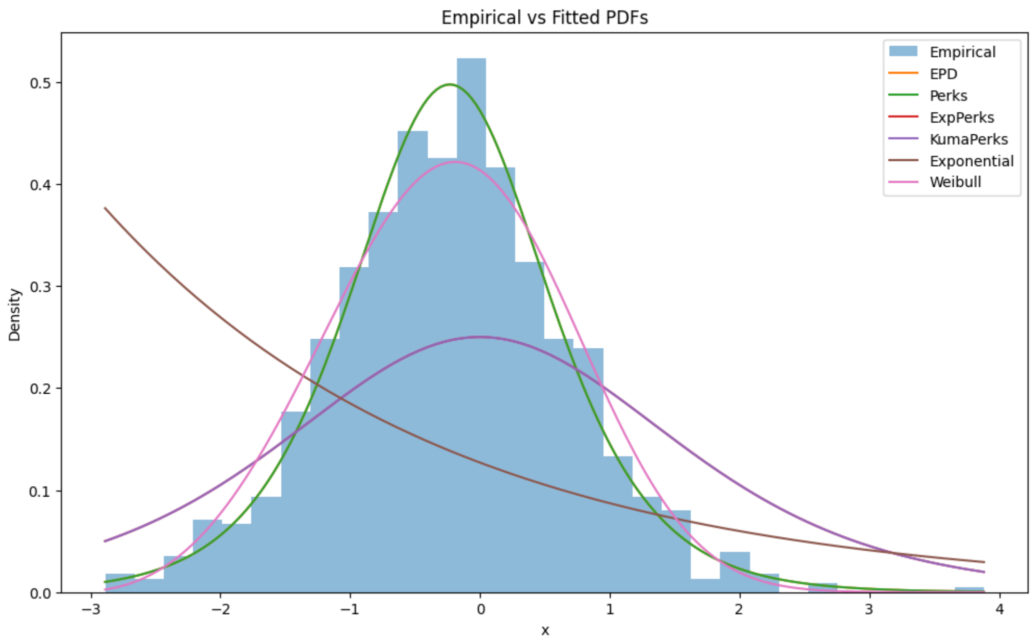
Model, comparison summary for COVID-19 Deaths Data

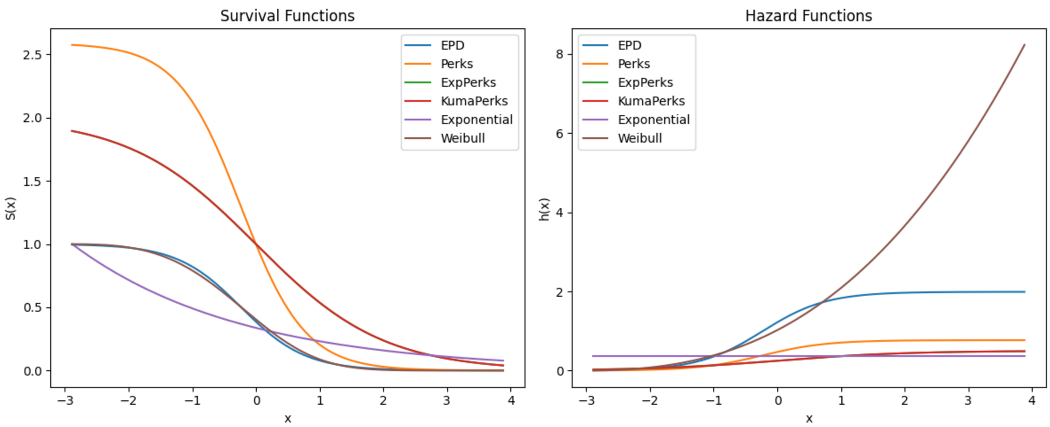
RANK	DISTRIBUTION	AIC	BIC	KS-STAT	W2	A2
1	EPD	651.9945	658.5634	0.0891	0.1112	0.7524
2	TLW	652.9520	659.5209	0.0905	0.1195	0.7835
3	KUMW	654.8269	663.5856	0.0915	0.1127	0.7650
4	EXPD	655.0399	663.7985	0.0916	0.1065	0.7490
5	CPD	1252.4014	1258.9703	0.2780	1.7790	12.4297

The above table presents the goodness-of-fit measures for competing models from the COVID-19 Deaths Data. The performance metrics used to assess the fit include

AIC, BIC, KS Stat, W2 and A2. The distribution model with the lowestvalues for these metrics indicates the best fit. In this case, the Exponential PerksDistribution (EPD) demonstratessuperior performance compared to the othermodels considered in the dataset.

PLOT OF EPD





SUMMARY AND CONCLUSION

This studycomprisesfivechapters thatcollectivelyinvestigatethe development,properties, and applicationsoftheExponentialPerksDistribution(EPD).ChapterOneintroducestheresearchbypresentingthebackgroundofprobabilitydistributiontheory,thelimitationsofexistingdistributions,andtherationale for extendingthe Perksdistributionthroughthe Exponential-Gfamily.Itclearlystates theresearchproblem,aim,objectives,andsignificanceofthestudy.ChapterTwoprovidesacomprehensivereviewofrelevantliterature,focusingoncontinuousdistributionfamilies,exponentialfamilyex tensions,andexistinggeneralizationsofthePerksdistribution.Thisreviewestablishesthetheoretical foundationandidentifiesresearchgapsthatthisworkaddresses.

CONCLUSION

ThisresearchsuccessfullydevelopstheExponentialPerksDistribution(EPD)asaflexibleextensionofthePerksdistributionthroughtheExponential-Gfamily.Theanalyticalderivations confirmthatthe EPDpossessesdesirablestatistical properties, includingaclosed-formquantile functionthat facilitatesr and omnumber generation, explicit expressions formoments andgeneratingfunctions,andasimplehazardfunctionthatexhibitsincreasinganddecreasingbehavior.Simulationresultsdemonstratethatmaximumlikelihoodestimatorsfor EPDparametersexhibitconsistency,withbiasandvariancedecreasingassamplesizesincrease.Theestimatorsshowreasonable precisionforsamplesizesof100ormoreandrobustnessacrossdifferentparameterconfigurations.Empiricalapplicationstoreal-worlddataconfirmtheEPD'spracticalutility.IntheCOVID-19mortality dataset,theEPDoutperformsfourcompetingdistributionswiththelowestAIC(651.9945)andadequategoodness-of-fit(KSp-value=0.6707).Forthebloodcancersurvivaldata set, the EPDagaindemonstratessuperiorfitwiththelowestAIC(656.4427)andexcellentgoodness-of-fit(KSp-value=0.9862).TheseresultsvalidatetheEPD'sflexibilityinmodelingdatasetswithvarying characteristics—fromtheright-skewedCOVID-19data(skewness=0.8365)totheslightlyleft-skewedbloodcancerdata(skewness=-0.4423).Thestudyconfirmsthatthe EPDprovidesastatisticallyrigorousandpracticallyusefulalternative toexistingdistributions for modeling positive real-valued data inmedicalandepidemiologicalcontexts.

REFRENCES

1. Adegoke, T., Yahya,W., and Adegoke, G. (2018). A study of the bayes estimator of the parameters in an inverted generalized exponential distribution with application. *Ann ≪ls Of St ≪tistic ≪l Theory And Applic ≪tions*, 1:1–10.
2. Alizadeh, M., Cordeiro, G. M., Pinho, L. G. B., and Ghosh, I. (2017). The gompertz-g family of distributions. *Journal of statistical theory and practice*, 11:179–207.
3. Alshanbari, H. M., Odhah, O. H., Ahmad, Z., Khan, F., and El-Bagoury, A. A.- H. (2023). A new probability distribution: model, theory and analyzing the recovery time data. *Axioms*, 12(5):477.
4. Bourguignon, M., Silva, R. B., and Cordeiro, G. M. (2014). The weibull-g family of probability distributions. *Journal of data science*, 12(1):53–68.

5. Chaudhary, A. K. and Kumar, V. (2013). A bayesian analysis of perks distribution via markov chain monte carlo simulation. *Nepal Journal of Science and Technology*, 14(1):153–166.
6. Godfrey Ieren, T., Chama, F., Alao Bamigbala, O., Joel, J., M Kromtit, F., Boyle Eraikhuemen, I., et al. (2020). On a shape parameter of gompertzinverse exponential distribution using classical and non classical methodsof estimation. *Journal of Scientific Research and Reports*, 25(6):1–10.
7. Gupta, R. D. and Kundu, D. (1999). Theory & methods: Generalized exponential distributions. *Australian & New Zealand Journal of Statistics*, 41(2):173–188.
8. Hayes, P. (2024). *Towards More Data Efficient Deep Learning*. PhD thesis, UCL(University College London).
9. Ieren, T., Oyamakin, S., and Chukwu, A. (2018). Modeling lifetime data withweibull-lindley distribution. *Biometrics and Biostatistics International Journal*, 7(6):532–544.
10. 9. Ieren, T. G. and Balogun, O. S. (2021). Exponential-lindley distribution: Theory
11. and application to bladder cancer data. *Journal of Applied Probability and Statistics*, 16(2):129–146.26
12. Ieren, T. G., Koleoso, P. O., Chama, A. F., Eraikhuemen, I. B., and Yakubu, N (2019). A lomax-inverse lindley distribution: Model, properties and applications to lifetime data. *Journal of Advances in Mathematics and Computer Science*, 34(3):1–28.
13. Ieren, T. G. and Yahaya, A. (2017). The weimal distribution: its properties and applications. *Journal of the Nigeria Association of Mathematical Physics*, 39:135–148.
14. Koleoso, P., Chukwu, A., and Bamiduro, T. (2019). A three-parameter gompertzlindley
15. distribution: Its properties and applications. *Journal of Mathematical Theory and Modeling*, 9(4)
16. Oguntunde, P. E., Khaleel, M. A., Okagbue, H. I., Opanuga, A. A., and Ilori, K. A.(2018b). Introducing the kumaraswamy perks distribution. In *Proceedingsof the world congress on engineering and computer science*, volume 2,pages 2078–0966
17. Perks, W. (1932). On some experiments in the graduation of mortality statistics. *Journal of the Institute of Actuaries*, 63(1):12–57.
18. Singh, B. and Choudhary, N. (2017). The exponentiated perks distribution. *International Journal of System Assurance Engineering and Management*, 8:468–478.